



## Tech Note #6: Calculating the Axis Level Closed Loop Gains

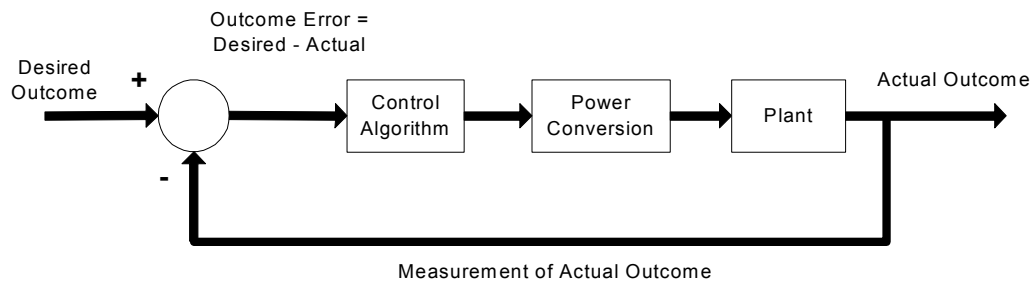
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### Abstract:

In this Tech Note the math required to allow the closed loop servo loop gains to be calculated as a function of the system model, and the desired frequency response will be provided and discussed. In addition, the equivalent continuous transfer function of the digital PID filter will be provided allowing for system simulation. Parts of this tech note will assume some prior knowledge in control system terminology.

### Closed Loop Control:

Closed loop control is the act of producing a control signal that will reduce the difference between some desired outcome and the current outcome as derived from a measurement.



### Defining the components in a closed loop servo axis

#### Control Variables:

For a servo system there are three outcomes, namely axis position, velocity, and torque.

#### Feedback:

The measurement of position comes from the incremental change in the axis's encoder from some defined starting point.

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The velocity measurement comes from the difference of the current position and previous position divided by the sample time of the control loop.

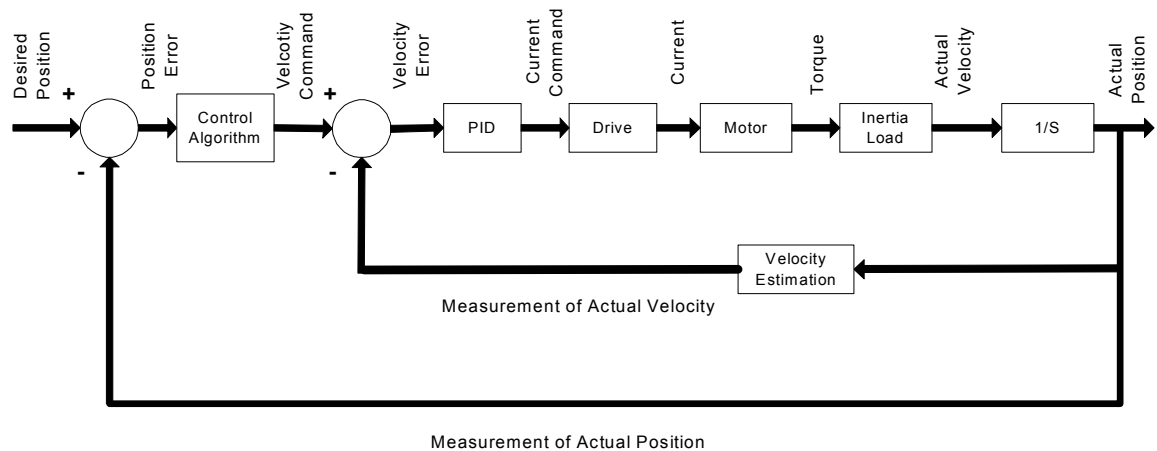
The torque is a calculated measurement from the current output of the servo drive. The torque of the motor is proportional to the current produced by the drive and the motor torque constant  $K_t$ .

#### Control Signal:

The produced control signal is an analog signal that is proportional to the required torque to reduce the difference in the control variables at a rate defined by the axis tuning.

#### Cascaded Control:

All of the I<sup>2</sup>T lines of motion controllers use a cascaded control paradigm. For a servo axis, the controller will close two loops, with one wrapped around another. The inside loop regulates the velocity of the axis while the outside loop regulates the position of the axis. There is actually a third loop in this structure, referred to as the current loop, however this is performed at the servo drive level.



Both the position, and the velocity loops use a PI filter. A PI filter is a Proportional – Integral filter. The proportional gain will provide system stiffness and response while the integral action assures that the systems achieves a no following error once the system has had time to settle.

### **System Modeling:**

In order to calculate the Axis level gains, a model of the system is required. This section will show how to calculate the individual components.

The servo system consists of the motion controller digital filters, the servo drive, motor, load, and feedback.

The model provided here is a simplified model, as it will consider time constants for the drive, and the electrical time constant for the motor, to be low enough, relative to the mechanical system's time constants, to be approximated as a only a gain, and not a filter.

### **Motion Controllers Sample Time:**

The I2T motion controllers use a 1 KHz scan rate for the closed loop filters.

$$T = .001 \text{ Sec.}$$

### **Motor:**

The motor will have a torque constant,  $K_t$ , listed on its nameplate. This torque constant may have units of lb\*in / amp, or N\*m / amp. For this discussion the units should be lb\*in / amp.

For this example we will use the following I<sup>2</sup>T Motor:

I2T-BM75E

Which has the following properties:

Inertia:

$$J_{\text{MOTOR}} = 0.4375 \text{ lbf}\cdot\text{in}\cdot\text{sec}^2 * 10^{-4}$$

Torque Constant:

$$K_t = .55 \text{ lbf}\cdot\text{in} / \text{amp}$$

### **Drive:**

I2T – 10 – 160 - S

10 Amps for 10 Volt input:

$$K_a = 1 \text{ Amps} / \text{Volt}$$

**Encoder:**

The encoder produces N pulses per revolution. The I2T motion controllers require the encoder to be a quadrature encoder, which means it outputs two signals, Channel A and B. Due to the quadrature relationship between these two channels the position resolution of the axis is 4 times the number of encoder lines.

So, for our example the encoder produces  
 1000 lines/revolution  
 or  
 4000 cnts/revolution

For our model we want the encoder gain to be in the units of cnts/rad, so we divide the encoder resolution by  $2\pi$ .

$$K_e = 4 * (2048) / 2\pi$$

$$K_e = 636.64 \text{ cnts / rad}$$

**DAC:**

The DAC or D-to-A converter converts 16-bit number to a signed analog voltage. So, for the total signed range of -10 to 10 volts the DAC system gain is calculated with:

$$K_c = 20 \text{ volts} / 65536 \text{ counts}$$

$$K_c = .0003 \text{ volts/counts}$$

**Total Inertia:**

The load inertia of the system must be added the motor inertia to obtain the total system inertia. For this example it is assumed that the coupling between the load and the motor is stiff. Compliance in the coupling between the load and motor can cause resonance, which may result in damage to the mechanical system.

For this example the load is just an aluminum disc with the following inertia:

$$J_{\text{load}} = 4.5154 \text{ lbf}\cdot\text{in}\cdot\text{sec}^2 * 10^{-4}$$

Adding this to the motor inertia gives us the system inertia:

$$J_{\text{tot}} = 4.9529 \text{ lbf}\cdot\text{in}\cdot\text{sec}^2 * 10^{-4}$$

### Calculating Gains:

Now that all of the model components are defined we can now estimate the controller gains to achieve a desired frequency response.

### Velocity Loop Bandwidth:

The velocity loop proportional gain is given by the following equation:

$$KVP := \frac{F_{vc} 2\pi \cdot J_{Total}}{K_t}$$

The units of this gain will be in amp / (rad/sec).

To convert this value to a value that should be entered into the motion controller the other system gains such as the encoder resolution, current gain, and DAC gain must be accounted for. So the KVP value to be entered into the controller for a given frequency response,  $F_{vc}$ , is calculated using the following equation:

$$KVP := \frac{F_{vc} 2\pi \cdot J_{Total}}{T \cdot K_a \cdot K_c \cdot K_e \cdot K_t}$$

Where,

- $F_{vc}$  (Hz) is the desired unity gain open-loop crossover frequency of the idealized rigid single mass system
- $J_{Total}$  (lb\*in\*sec<sup>2</sup>) is the total system inertia
- $K_t$  (lb \* in/ amp) is the motor's torque constant
- $K_e$  (cnts / rad) is the motor's encoder gain
- $K_c$  (counts / volt) is the controller's DAC gain
- $K_a$  (amp/volt) is the drive's current gain
- $T$  (sec/tick) is the scan time of the PID loop

So, in our example a desired frequency response of 25 Hertz, which is a normal response, would result in the following gain for the controller:

$$KVP = 0.141 \text{ amp / (rad/sec)}$$

Or

$$KVP = 728$$

A typical servo system may be tuned to something like 30 Hertz. This requires a rigid mechanical coupling, or damage to the mechanical system may occur.

### Velocity Loop Integral Gain:

A general suggested setting for the frequency of the integral action is that it should equal to, or less than 0.1 times the cross over frequency used to calculate the velocity loop proportional gain.

For this example we will chose a setting of .025 Hertz.

$$F_{Ivc} = .025 \text{ hertz}$$

To convert this gain into a value to be used to model a continuous system use the following equation:

$$KVI := \frac{F_{Ivc}(2\pi)^2 \cdot J_{Total}}{K_t}$$

The units of this gain will be in amp / (rad/sec).

To convert this value to a value that should be entered into the motion controller the other system gains such as the encoder resolution, current gain, and DAC gain must be accounted for. So the KVP value to be entered into the controller for a given frequency response,  $F_{vc}$ , is calculated using the following equation:

$$KVI := \frac{F_{Ivc}(2\pi)^2 \cdot J_{Total}}{K_t \cdot K_a \cdot K_c \cdot K_e \cdot T}$$

Where,

- $F_{vc}$  (Hz) is the desired unity gain open-loop crossover frequency of the idealized rigid single mass system

So, in our example a desired integral frequency response of .025 Hertz would result in the following gain for the controller:

$$KVI = 0.00089 \text{ amp / (rad/sec)}$$

Or

$$KVI = 18736$$

### Position Loop Bandwidth:

The position loop gain determines the settling time of the position loop. The gain setting is calculated from a desired bandwidth of the position loop, assuming an ideal velocity loop. In general, the higher the position loop bandwidth, the faster the settling time. The actual performance of the position loop is dependent on the bandwidth of the velocity loop. So, choosing a position loop bandwidth that is 1/5<sup>th</sup> the velocity loop bandwidth is a good start. If position overshoot can be tolerated in the system then a higher bandwidth can be selected.

The Position loop proportional gain is given by the following equation:

$$KPP := F_{pc} 2\pi$$

The units of this gain will be in (1/sec).

To convert this value to a value that should be entered into the motion controller the other system gains such as the encoder resolution, current gain, and DAC gain must be accounted for. So the KVP value to be entered into the controller for a given frequency response,  $F_{vc}$ , is calculated using the following equation:

$$KPP := KPP \cdot T \cdot 2^{16}$$

Where,

- $F_{pc}$  (Hz) is the desired unity gain open-loop crossover frequency of the idealized rigid single mass system
- $T$  (sec/tick) is the scan time of the PID loop

So, in our example a desired frequency response of 5 Hertz, which is a Gentle response, would result in the following gain for the controller:

$$KPP = 31.4 \text{ (1/sec)}$$

Or

$$KPP = 2051$$

A typical servo system may be tuned to something like 20 Hertz in the position loop. This requires a rigid mechanical coupling, and a properly tuned velocity loop or damage to the mechanical system may occur.